Hasil Penelitian Hibah Fundamental "Sekitar Path Coalgebras dan Path Algebras"

On Path Coalgebras and Path Algebras

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EXTENDED ABSTRACT

In this research we study the relations between path coalgebras and path algebras. Algebras can be represented as quiver (directed graph) and modules can berepresented as quiver representation [ASS2006]. A quiver Q is a quadruple $(Q_0;Q_1; s; t)$ where Q_0 is the set of vertices (points), Q_1 is the set of arrows and for each arrow $\alpha \in Q_1$; the vertices $s(\alpha)$ and $t(\alpha)$ are the source and the target of α; respectively. If i and j are vertices, an (oriented) path in Q of length m from i to j is a formal composition of arrows $p = \alpha_1\alpha_2 \ldots \alpha_m$ where $s(\alpha_1) = i$, $t(\alpha_m) = j$ and $t(\alpha_{k-1}) = s(\alpha_k)$ for $k = 2,3,\ldots, m$.

Let KQ be the K-vector generated by the set of all paths in Q. Then KQ can be endowed with a structure of K-algebra with multiplication induced by concatenation of paths, KQ is called the path algebra of the quiver Q (see [M2005] and [M2009]). The path algebra KQ also can be viewed as a graded K-coalgebra. The coalgebra KQ is the path coalgebra of the quiver Q. For the convenience we denote by KQ the path algebra of Q and by CQ the path coalgebra of Q.

A basic K-algebra is isomorphic to the factor algebra of the path algebra KQ_A by an admissible ideal, where Q_A is the quiver of A. Since any finite dimensional algebra is Morita equivalent to a uniquely determined basic algebra, it follows that any finite dimensional algebra A over an algebraically closed field is Morita equivalent to KQ_A modulo an admissible ideal. In [An2009] we proved the following result in [ASS2006]: Given finite, a cyclic and connected quiver Q then the path algebra $A = KQ$ is hereditary and $Q = Q_A$. In [Mar2009]) we showed that if A is a hereditary , basic, connected algebra having a complete set primitive orthogonal idempotent then the quiver Q_A is finite,

connected and acyclic. We also showed that there is a K-algebra isomorphism between A and the path algebra KQ_A. This means that hereditary algebras are isomorphic to their path algebras.

In coalgebra representation theory, in [Chin2004] Chin explained how to represent coalgebras as quiver, and also how to construct coalgebras form quivers called path coalgebras. Moreover in [Chin2002] Chin showed that hereditary coalgebras are isomorphic to their path coalgebras. Some relations between hereditary algebras and coalgebras are discovered by Natasescu et al [NTZ96] and Garminia et al [GAI2007]. They showed that for a finite dimensional coalgebra C, then C is hereditary coalgebra if and only if its dual $D(C) = \mathrm{Hom}_K(C,K)$ is a hereditary algebra.

For a quiver Q, we denote by Q^{op} the quiver having the same set of vertices and reversing arrows. Let CQ be a finite dimensional path coalgebra, $D(CQ)$ be its dual and KQ be a finite dimensional path algebra, $D(KQ)$ be its dual. In [MG2009] we showed that the quiver of a path algebra (resp. path coalgebra) is the dual quiver of its path algebra (resp. algebra) and this result gives an alternative proof of the results in [NTZ96] and [GAI2007] for the basic and pointed cases. Moreover one can study more about algebras and coalgebras through their quivers. To obtain this result, first we showed that $D(KQ)$ is a path coalgebra and $D(CQ)$ is a path algebra, then we made one-to-one correspondences between $D(KQ)$ and $C(Q^{op})$ and also between $D(CQ)$ and $K(Q^{op})$.

In case CQ and KQ are not finite dimensional, we showed in [MGA2009] that the same results hold for bound path algebras KQ/I and bound path coalgebras $C(Q,I)$ where I is some admissible ideal. We show that the quiver of a bound path algebra (resp. bound path coalgebra) is the dual quiver of its bound path coalgebra (resp. bound path algebra), hence the dual of basic algebras are pointed coalgebras, and vice versa. To obtain this result, we proved that if I is an admissible ideal of KQ^{op} then $D(KQ^{op}/I)$ isomorphic to $C(Q, I)$, and if C is a finite dimensional relation subcoalgebra of CQ then there exists some admissible ideal I of KQ^{op} such that $D(C) = KQ^{op}/I$.

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[MGA2009] Muchtadi-Alamsyah,I., Garminia,H., Astuti,P. *Quivers of Bound Path Algebras and Bound Path Coalgebras,* submitted to ITB Journal of Sciences

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List of Research Output:

1. Anwar,F. *Hereditary Path Algebra,* Proceeding IndoMS International Conference on Mathematics and Its Applications, Jogjakarta, 2009.
2. Mardiyati, U. *Hereditary Path Algebra and Its Characteristic Trough Injective Module,* Proceeding IndoMS International Conference on Mathematics and Its Applications, Jogjakarta, 2009.

3. Muchtadi-Alamsyah,I. *Quiver sebagai Representasi Aljabar*, Proceeding Seminar Nasional Aljabar, Pengajaran, danTerapannya, Jogjakarta, January 2009.

4. Muchtadi-Alamsyah,I., Garminia,H., *Quivers of Path Algebras and Path Coalgebras,* Proceeding IndoMS International Conference on Mathematics and Its Applications, Jogjakarta, 2009.

5. Muchtadi-Alamsyah,I., Garminia,H., Astuti,P. *Quivers of Bound Path Algebras and Bound Path Coalgebras,* to be submitted to ITB Journal of Sciences