METAXIAL DIFFRACTION METHOD

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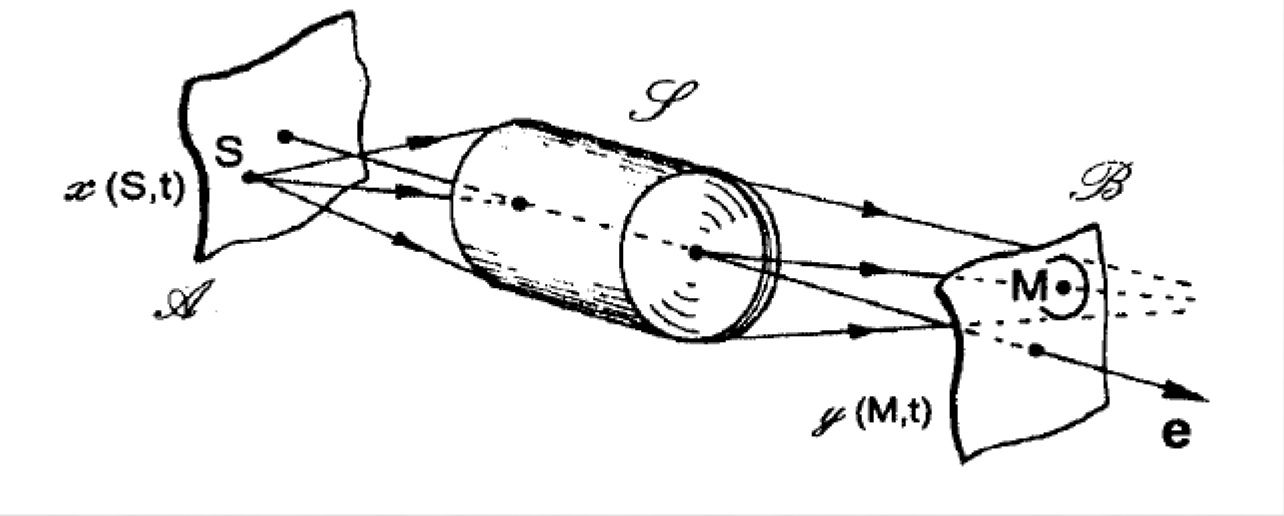
The Need of Unified Framework

- Optical communications is a must in broadband era.
- Optical communications can be free space or through optical fiber
- Fundamental fenomena: field transfer from transmitter to the receiver through electromagnetic system
- In this presentation, we propose unified framework to analyse and design of optical systems – Metaxial Optics-- developed by G. Bonnet and P. Pellat-Finet.

SPATIAL DOMAIN FIELD TRANSFER

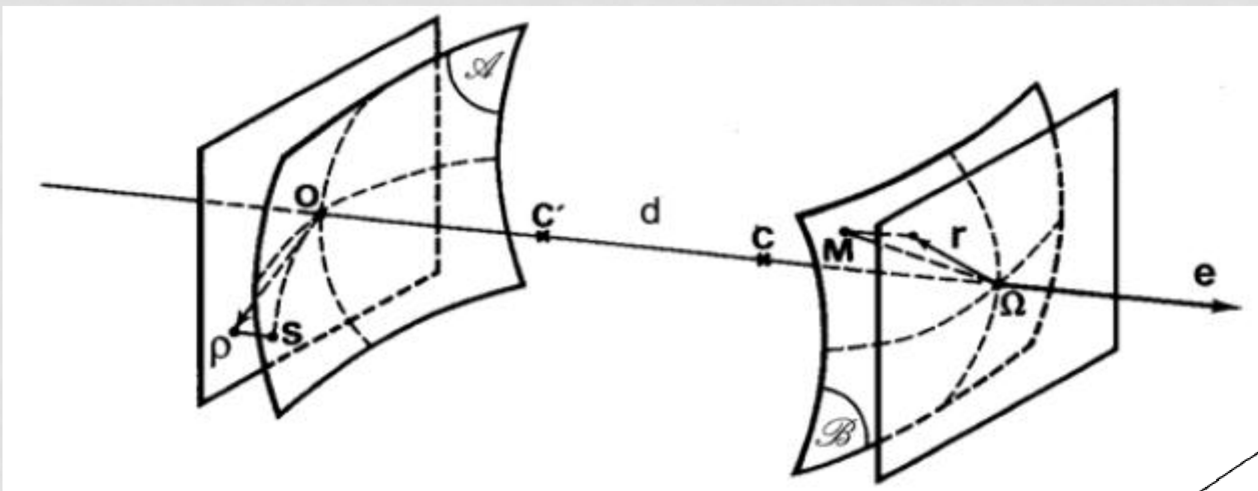
PRINCIPLES

- Radiative transfer between transmitter and receiver traversing an electromagnetic system
- Geometric : spherical transmitter, receiver, and wavefront
  - plane is a special case
- Field transfer system

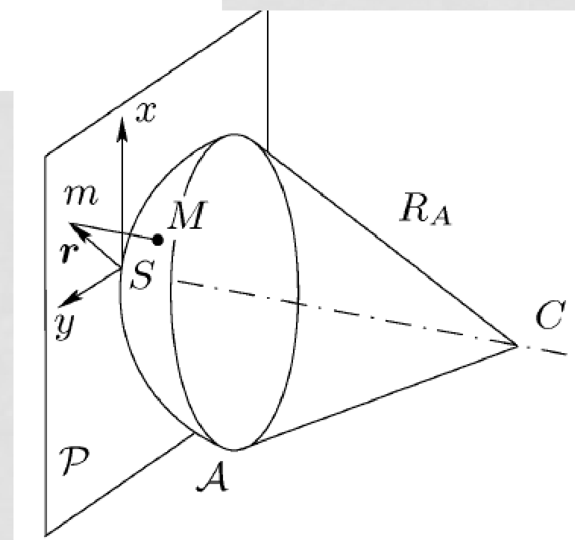


GENERAL SPATIAL PROBLEM

- Field transfer from spherical A to spherical B

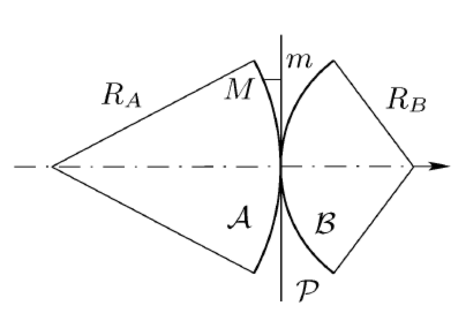


- Transmitter coordinate



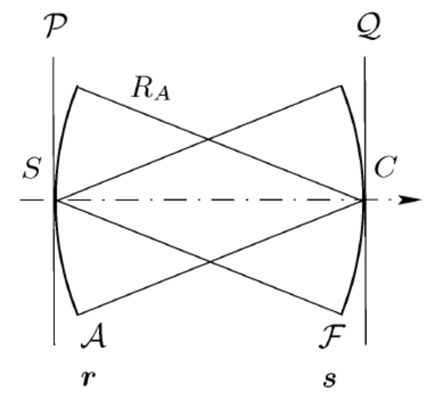
PHASE AND FOURIER TRANSFORMATIONS

- Phase transformation (curvature transparancy)
  - Field transfer between two tangential sphere



$$U_B(\boldsymbol{r}) = U_A(\boldsymbol{r}) \exp\left[-\frac{\mathrm{i}\pi}{\lambda}\left(\frac{1}{R_B} - \frac{1}{R_A}\right) r^2\right]$$

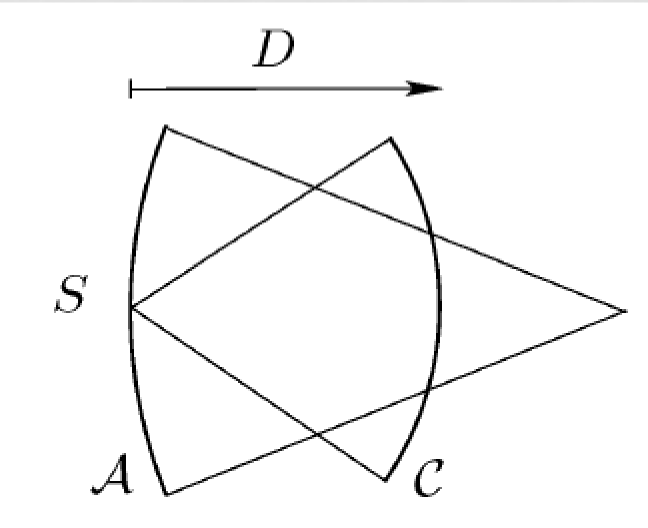
- Fourier transformation:
  - field transfer between two conjugate sphere



$$U_F(\boldsymbol{s}) = \frac{\mathrm{i}}{\lambda R_A} \widehat{U}_A\left(\frac{\boldsymbol{s}}{\lambda R_A}\right) = \frac{\mathrm{i}}{\lambda D} \widehat{U}_A\left(\frac{\boldsymbol{s}}{\lambda D}\right)$$

We omit propagation delay factor $\exp\left[-\frac{2\mathrm{i}\pi R_A}{\lambda}\right]$

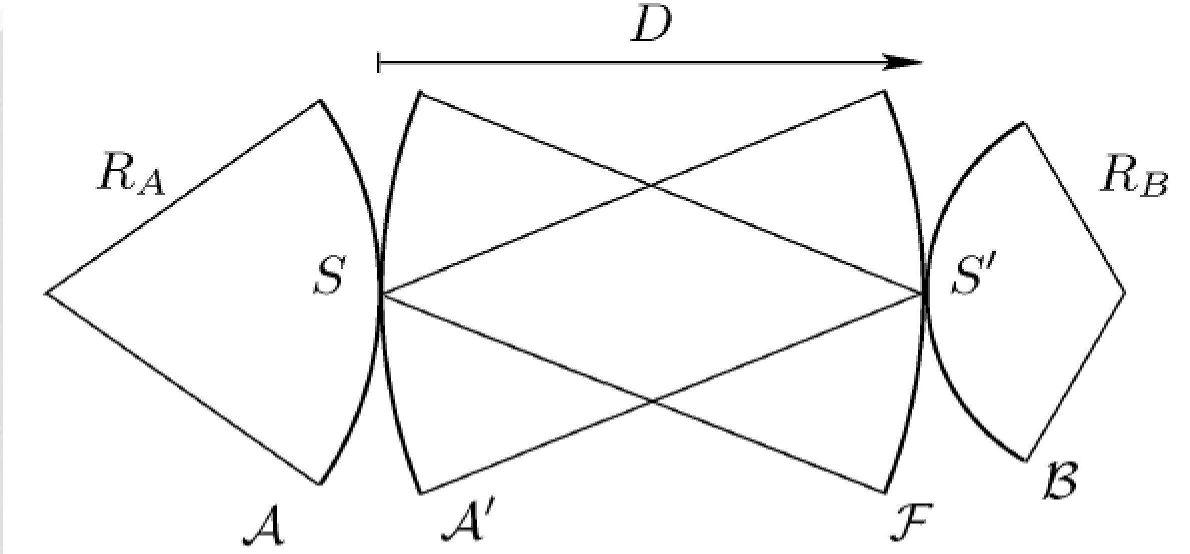
SPECIAL CASE: FRESNEL DIFFRACTION



Field transfer between cardinal sphere :
From sphere A to sphere C

$$U_C(\boldsymbol{s}) = \frac{\mathrm{i}}{\lambda D} \int_{\mathbb{R}^2} \exp\left[-\frac{\mathrm{i}\pi}{\lambda}\left(\frac{1}{D} - \frac{1}{R_A}\right) r^2\right] \exp\left[\frac{2\mathrm{i}\pi}{\lambda D}\, \boldsymbol{s}\cdot\boldsymbol{r}\right] U_A(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}$$

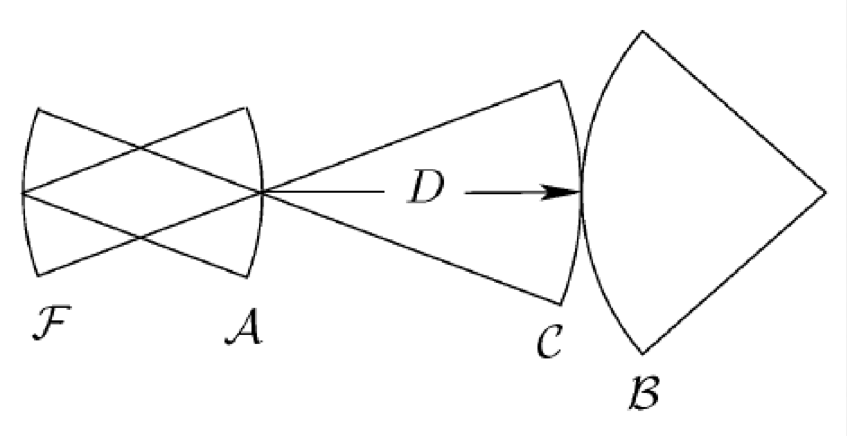
THE GENERAL TRANSFER



$$U_B(\boldsymbol{s}) = \frac{\mathrm{i}}{\lambda D} \exp\left[-\frac{\mathrm{i}\pi}{\lambda}\left(\frac{1}{R_B}+\frac{1}{D}\right) s^2\right] \times \int_{\mathbb{R}^2} \exp\left[-\frac{\mathrm{i}\pi}{\lambda}\left(\frac{1}{D}-\frac{1}{R_A}\right) r^2\right] \exp\left[\frac{2\mathrm{i}\pi}{\lambda D}\,\boldsymbol{s}\cdot\boldsymbol{r}\right] U_A(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}$$

THE BONNET TRILOGY : FRESNEL FILTERING

- A Fresnel diffraction phenomenon from an emitter to a receiver can be composed into three operations
  1. An optical (spatial) Fourier transform (Fraunhofer diffraction)
  2. A spatial filtering, between Fourier sphere and a cardinal sphere tangent to the receiver
  3. A phase transformation (curvature transparancy) between the cardinal sphere and the receiver

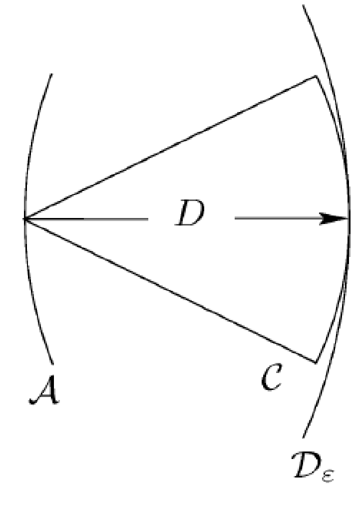


FRACTIONAL FOURIER:
GENERAL SPHERE TO SPHERE FIELD TRANSFER

- 2D Fractional Fourier Transformation Definition

$$\mathcal{F}_\alpha[f](\boldsymbol{\sigma}) = \frac{\mathrm{i}e^{-\mathrm{i}\alpha}}{\sin\alpha}\exp[-\mathrm{i}\pi\sigma^2\cot\alpha] \times \int_{\mathbb{R}^2}\exp[-\mathrm{i}\pi\rho^2\cot\alpha]\exp\left[\frac{2\mathrm{i}\pi\boldsymbol{\sigma}\cdot\boldsymbol{\rho}}{\sin\alpha}\right] f(\boldsymbol{\rho})\,\mathrm{d}\boldsymbol{\rho}$$

- Metaxial interpretation $\varepsilon R_A > 0$ α in $]-\pi,\pi[$



$$\cot\alpha = \varepsilon\frac{1-\mu}{\mu}, \quad \alpha D \geq 0$$

$$V_A(\boldsymbol{\rho}) = U_A\left(\sqrt{\lambda\varepsilon R_A}\,\boldsymbol{\rho}\right),$$

$$V_C(\boldsymbol{\sigma}) = U_C\left(\sqrt{\lambda\varepsilon R_A}\frac{\boldsymbol{\sigma}}{\cos\alpha+\varepsilon\sin\alpha}\right)$$

$$\boldsymbol{\rho} = \frac{1}{\sqrt{\lambda\varepsilon R_A}}\boldsymbol{r},$$

$$\boldsymbol{\sigma} = \frac{1}{\sqrt{\lambda\varepsilon R_A}}(\cos\alpha+\varepsilon\sin\alpha)\boldsymbol{s}$$

$$R_\varepsilon = \frac{\mu^2+\varepsilon^2(1-\mu)^2}{-\mu+\varepsilon^2(1-\mu)}R_A = \frac{D^2+\varepsilon^2(R_A-D)^2}{-D+\varepsilon^2(R_A-D)}$$

$$V_{D\varepsilon}(\boldsymbol{\sigma}) = e^{\mathrm{i}\alpha}(\cos\alpha+\varepsilon\sin\alpha)\,\mathcal{F}_\alpha[V_A](\boldsymbol{\sigma})$$

TIME DOMAIN FIELD TRANSFER SECOND ORDER DISPERSIVE OPTICAL FIBER CHANNEL

FIELD PROPAGATION ALONG THE LINE

- Field representasion

$$\begin{tikzcd}
& E(z,t) \arrow[dl, shift left, harpoon, "z"'] \arrow[dr, shift left, harpoon, "t"] & \\
\mathfrak{e}(f,t) \arrow[ur, shift left, harpoon] \arrow[dr, shift left, harpoon, "t"'] & & e(z,\nu) \arrow[ul, shift left, harpoon] \arrow[dl, shift left, harpoon] \\
& \mathfrak{E}(f,\nu) \arrow[ul, shift left, harpoon] \arrow[ur, shift left, harpoon, "z"'] &
\end{tikzcd}$$

$$\mathfrak{e}(f,t) = \int_{\mathbb{R}} E(z,t)\,\mathrm{e}^{2\mathrm{i}\pi f z}\,\mathrm{d}z$$

$$E(z,t) = \int_{\mathbb{R}} \mathfrak{e}(f,t)\,\mathrm{e}^{-2\mathrm{i}\pi f z}\,\mathrm{d}f$$

$$e(z,\nu) = \int_{\mathbb{R}} E(z,t)\,\mathrm{e}^{-2\mathrm{i}\pi \nu t}\,\mathrm{d}t$$

$$E(z,t) = \int_{\mathbb{R}} e(z,\nu)\,\mathrm{e}^{2\mathrm{i}\pi \nu t}\,\mathrm{d}\nu$$

WAVE PACKET PROPAGATION

- We choose spatial (longitudinal) parameter $\mathfrak{e}(f,t) = \mathfrak{a}(f)\,\mathrm{e}^{2\mathrm{i}\pi\nu t}$
- Then $$E(z,t) = \int_{\mathbb{R}} \mathfrak{a}(f)\,\mathrm{e}^{2\mathrm{i}\pi\nu t}\,\mathrm{e}^{-2\mathrm{i}\pi f z}\,\mathrm{d}f$$
- Because we choose spatial frequency as a parameter, then the (temporal) frequency is a function of f $$E(z,t) = \int_{\mathbb{R}} \mathfrak{a}(f)\,\mathrm{e}^{2\mathrm{i}\pi[\nu(f)t - f z]}\,\mathrm{d}f$$
- We choose a wave packet function a defined by
- $$a(\nu) = \mathfrak{a}[f(\nu)]\,\frac{\mathrm{d}f}{\mathrm{d}\nu}$$
- Then $$E(z,t) = \int_{\mathbb{R}} a(\nu)\,\mathrm{e}^{2\mathrm{i}\pi[\nu t - f(\nu) z]}\,\mathrm{d}\nu$$

NARROW BAND PROPAGATION

- For narrow-band signal

$$\nu(f) = \widetilde{\nu} + (f - \widetilde{f}\,)\frac{\mathrm{d}\nu}{\mathrm{d}f}(\widetilde{f}\,) + \frac{1}{2}(f - \widetilde{f}\,)^2\frac{\mathrm{d}^2\nu}{\mathrm{d}f^2}(\widetilde{f}\,) + \ldots$$

- With the group velocity $V_{\mathrm{g}} = \frac{\mathrm{d}\nu}{\mathrm{d}f}(\widetilde{f}\,)$

- Narrow band propagation (the signal is delayed alang the line)

$$\begin{aligned} E(z,t) &= \mathrm{e}^{2\mathrm{i}\pi(\widetilde{\nu}t - \widetilde{f}V_{\mathrm{g}}t)} \int_{\mathbb{R}} \mathfrak{a}(f)\, \mathrm{e}^{2\mathrm{i}\pi f(V_{\mathrm{g}}t - z)}\, \mathrm{d}f \\ &= \mathrm{e}^{2\mathrm{i}\pi(\widetilde{\nu}t - \widetilde{f}V_{\mathrm{g}}t)}\, E\left(z - V_{\mathrm{g}}t, 0\right) . \end{aligned}$$

LINEAR FREQUENCY MODULATION

SECOND ORDER DISPERSIVE OPTICAL FIBER

- In linear frequency modulation, we have $\nu = \nu_0 + \nu_0 \theta t$

$$U(t) = \mathrm{e}^{2\mathrm{i}\pi\nu t} = \mathrm{e}^{2\mathrm{i}\pi\nu_0 t}\mathrm{e}^{2\mathrm{i}\pi\nu_0\theta t^2} = U_0(t)\,\mathrm{e}^{2\mathrm{i}\pi\nu_0\theta t^2}$$

$$e_z(\nu) = e_0(\nu)\,\mathrm{e}^{-\mathrm{i}\beta(\nu)z}$$

- Limited to second order, with $\beta_1 = (\mathrm{d}\beta/\mathrm{d}\nu)(\widetilde{\nu})$ $\beta_2 = (\mathrm{d}^2\beta/\mathrm{d}\nu^2)(\widetilde{\nu})$
- $\beta(\nu) = \beta_0 + \beta_1(\nu - \widetilde{\nu}) + \frac{\beta_2}{2}(\nu - \widetilde{\nu})^2$

$$E_z(t) = \mathrm{e}^{-\mathrm{i}(\beta_0 - \beta_1\widetilde{\nu})z} \int_{\mathbb{R}} E_0(t')\sqrt{\frac{2\pi}{|\beta_2 z|}}\,\mathrm{e}^{-\mathrm{i}\mathfrak{s}(\beta_2 z)\pi/4}$$
$$\times \;\; \exp\left[2\mathrm{i}\pi\widetilde{\nu}\left(t - t' - \frac{\beta_1}{2\pi}z\right)\right] \exp\left[\frac{2\mathrm{i}\pi^2}{\beta_2 z}\left(t - t' - \frac{\beta_1}{2\pi}z\right)^2\right]\,\mathrm{d}t'$$
$$= \mathrm{e}^{-\mathrm{i}\beta_0 z}\int_{\mathbb{R}} E_0(t')\sqrt{\frac{2\pi}{|\beta_2 z|}}\,\mathrm{e}^{-\mathrm{i}\mathfrak{s}(\beta_2 z)\pi/4}\exp[2\mathrm{i}\pi\widetilde{\nu}(t - t')]$$
$$\times \;\; \exp\left[\frac{2\mathrm{i}\pi^2}{\beta_2 z}\left(t - t' - \frac{\beta_1}{2\pi}z\right)^2\right]\,\mathrm{d}t'\,.$$

The expression is very similar to Fresnel diffraction

COMPLEX ENVELOPE

$$A_z(t) = \mathrm{e}^{-\mathrm{i}\beta_0 z}\sqrt{\frac{2\pi}{|\beta_2 z|}}\,\mathrm{e}^{-\mathrm{i}\mathfrak{s}(\beta_2 z)\pi/4}\exp\left[\frac{2\mathrm{i}\pi^2 t^2}{\beta_2 z}\right]$$
$$\times\int_{\mathbb{R}} A_0(t')\exp\left[\frac{2\mathrm{i}\pi^2}{\beta_2 z}\left(t'+\frac{\beta_1}{2\pi}z\right)^2\right]\exp\left[-\frac{4\mathrm{i}\pi^2 t}{\beta_2 z}\left(t'+\frac{\beta_1}{2\pi}z\right)\right]\mathrm{d}t'\,.$$

- We define complex envelope function:

$$B_z(t) = \sqrt{\frac{2\pi}{|\beta_2 z|}}\,\mathrm{e}^{-\mathrm{i}\mathfrak{s}(\beta_2 z)\pi/4}\int_{\mathbb{R}} A_0(t')\,\exp\left[\frac{2\mathrm{i}\pi^2}{\beta_2 z}(t-t')^2\right]\mathrm{d}t'$$

$$B_z(t) = \sqrt{\frac{2\pi}{|\beta_2 z|}}\,\mathrm{e}^{-\mathrm{i}\mathfrak{s}(\beta_2 z)\pi/4}\,\exp\left[\frac{2\mathrm{i}\pi^2 t^2}{\beta_2 z}\right]$$
$$\times\int_{\mathbb{R}} B_0(t')\,\exp\left[\frac{2\mathrm{i}\pi^2 t'^2}{\beta_2 z}\right]\,\exp\left[-\frac{4\mathrm{i}\pi^2 t t'}{\beta_2 z}\right]\mathrm{d}t'$$

ENVELOPE COMPLEX PROPAGATION

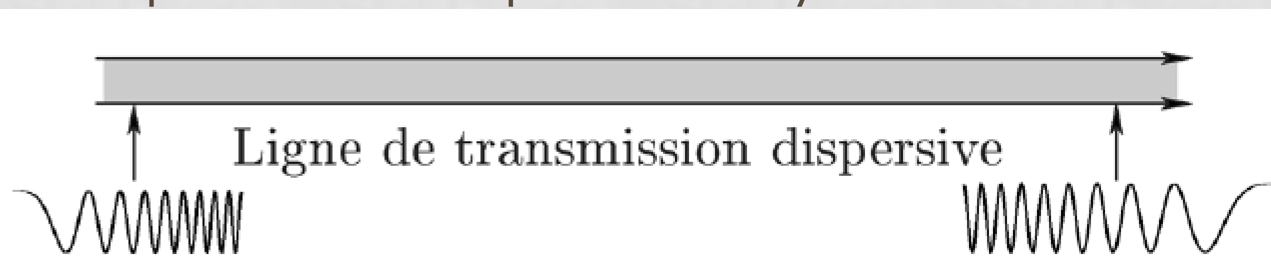
- Define linear frequency modulation function

$$T(t) = \mathrm{e}^{2\mathrm{i}\pi\widetilde{\nu}\theta t^2}$$

$$\mathit{\Lambda} = \frac{\mathfrak{s}(\beta_2)}{\widetilde{\nu}} \qquad R = \frac{\mathfrak{s}(\beta_2)}{2\theta} \qquad \mathit{\Lambda} R = \frac{1}{2\widetilde{\nu}\theta}$$

$$T(t) = \exp\left[\frac{\mathrm{i}\pi}{\mathit{\Lambda} R} t^2\right]$$

- Complex envelope linearly modulated



$$Z = \frac{\beta_2 z}{2\pi\mathit{\Lambda}}$$

$$U_0(t) = B_0(t)\,\exp\left[\frac{\mathrm{i}\pi t^2}{\mathit{\Lambda} R_0}\right] \qquad U_z(t) = B_z(t)\,\exp\left[\frac{\mathrm{i}\pi t^2}{\mathit{\Lambda} R_z}\right]$$

FRACTIONAL FOURIER TRANSFORM
REDUCED ENVELOPE COMPLEX PROPAGATION

- We define reduced envelope complex

$$V_0(\tau') = U_0(\sqrt{\varepsilon \Lambda R_0}\, \tau')\,,$$

$$V_z(\tau) = U_z\left(\frac{\sqrt{\varepsilon \Lambda R_0}}{\cos\alpha + \varepsilon \sin\alpha}\,\tau\right)\,.$$

$$V_z(\tau) = \mathrm{e}^{\mathrm{i}\alpha/2}\sqrt{\cos\alpha + \varepsilon\sin\alpha}\;\exp\left[\mathrm{i}\pi\left(\frac{1}{R_z} + \frac{1}{Z}\right)\frac{\varepsilon \Lambda R_0 \tau^2}{(\cos\alpha + \varepsilon\sin\alpha)^2}\right] \times \int_{\mathbb{R}} \exp[\mathrm{i}\pi\tau'^2 \cot\alpha]\,\exp\left[-\frac{2\mathrm{i}\pi\tau\tau'}{\sin\alpha}\right] V_0(\tau')\,\mathrm{d}\tau'\,.$$

- This is fractional Fourier transform if $\left(\frac{1}{R_z} + \frac{1}{Z}\right)\frac{\varepsilon \Lambda R_0}{(\cos\alpha + \varepsilon\sin\alpha)^2} = \cot\alpha$

$$V_z(\tau) = \mathrm{e}^{\mathrm{i}\alpha/2}\sqrt{\cos\alpha + \varepsilon\sin\alpha}\,\mathcal{F}_\alpha[V_0](\tau)$$

REFERENCES

1. Pierre Pellat-Finet, OPTIQUE DE FOURIER : Theorie Metaxial et Fractionnaire, Springer-Verlag France, 2009
2. Georges Bonnet, OPTIQUE METAXIAL (phenomenologie), polycopie, GESSY – Universite du Sud