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# Detailed Critical Numerical Study of Discretizing Effects in Optimizing Using Bellman's Dynamic Programming Method

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ABSTRACT

A detailed numerical study using the concrete example of the Citarum reservoir management in Indonesia is acheived to determine the effect of the discretization of time and state function (water stock volume). The influence of variations of parameters $\Delta t$ and $\Delta\Omega$ and on the graph which gives the maximum output relating to stock initial state and on optimal way track which defines the best management is examined for different hypotheses on input estimation and utility function definition. From the sensitivity of the maximum value of economic function, the exploitation possibility to acheive a result corresponding to a thin cut from quite rough discretization is examined. Instability risks are detected for quite rough cut. In every case, the correlation of time step and space step are scrutinized. Incertanties, errors, due to the cuts are compared with unavoidable data uncertanties and peculiarly with climate randomness.

## 1. INTRODUCTION

The importance of research into the optimal solution to the employment of water resources cannot be over emphasised. This is a reality which is becoming highly topical even in the high income states because of the warning dryness and the increase of expensive water consumation in the under developed states, where water is precisely one of the first factors in social and economic progress.

Certainly, optimization often involves heavy investment in water exploitation and the realisation of the presented equipment as well as in management. We frequently try to optimize only the exploitation as the equipment exists already. It is from this point of view that we present what is going to follow. It is as well to lighten the presentation to best focus our objectives, which remain academic though leaning on a concrete example.

To illustrate our subject, instead of working on utopic data, we have chosen to use the real example of the Citarum reservoir on Java Island in Indonesia. The optimization study of reservoir management to chain goals will be executed with the dynamic programming illustrated by the magnificent Bellman's Method. As is shown by the title, our subject is a detailed numerical critical study of the discretization effect of essential continuous variables, which are the time and the water stock volume, which constitute the state function of the simple considered system.

We will begin with a brief description of the site and then we will present the obtained results in the optimization by considering time steps decreasing from a quarter to fifteen days under the debatable hypothesis of one year periodically repeatable average hydrologic.

The precision and the stability of optimal results will also be examined for different steps of discretization of volume which go from the third to the sixtieth part effective storage capacity of the reservoir with some intermediary values.

## 2. PRESENTATION OF THE INTEGRATED INSTALLATIONS HYDROELECTRIC POWER STATION IN THE RIVER CITARUM BASIN

The Citarum basin presented on the map in Figure 1 is situated in the equatorial tropical zone of the latitude 06° 3' to the latitude 07° 17' south, it is included between the meridians 106° 8' and 108° 4' east. It is bound by two types of monsoon: from Australia and from Asia. The oriental monsoon of Australia blows from April to September, that is the dry season. The western monsoon of Asia services from October to March, that is the wet season.

The Citarum river flows northward to the Java sea through the central part of west Java from its sources in the mountainous area surrounding Bandung City. This basin is surrounded by high mountains, in the north by Tangkuban Perahu (+2,276 m), in the south-east by Guntur (+2,249 m) and in the south by Patuha (+2,434 m). Its catchment area of 6,000 $km^2$ is the largest among the rivers in west Java and ranks third among the main rivers on Java Island; it is the most important in Indonesia from the point of view of energy. The total length of the main river is approximately 350km. There exist three sub-basins: the upper basin of Saguling, the middle basin of Cirata and the lowest basin of Jatiluhur. Dams were constructed on these three cascades to use the water of the Citarum basin for hydroelectric power.

### Cenral Hydroelectric Power Station of Saguling.

This hydroelectric construction exploits the upper cascade of Saguling with a catchment area of 2,283 $km^2$ and an average annual discharge of 81.4 $m^3/s$ with an observation period of 60 years (3). This construction has a rockfill dam type with the height of 99 m, the crest length of 301.4 m and the volume of 2.7 $hm^3$. The stocking capacity maximum of the reservoir is 600 $hm^3$ with a high water level of 643 m, a total submerged area of 53.4 $km^2$, and a low water level of 623 m, while the maximum flood water level is 645 m, and the maximum total submerged area is 56.1 $km^2$. The installed capacity of the power station is (4x175) MW, its maximum discharge 228 $m^3/s$, its maximum effective head 362.4 m and the tailrace waterlevel is 252 m. The annual generated energy is 2,156 GWh. This hydroelectric power has been in service since 1986.

**Cirata Hydroelectric Power Project.**

This project exploits a cascade situated between the upper basin of Saguling and the lowest basin of Jatiluhur. The catchment area of Cirata is 4,060 $km^2$ and its average annual discharge is 171 $m^3/s$ with an observation period from 1920 to 1980. A concrete faced rockfill dam is built on the cascade of Cirata. The height of the dam is 125 m and it can be increased up to 140 m on the second stage. The hydroelectric power station is situated underground. This hydroelectric power has been in service since 1989.

**Jatiluhur Hydroelectric Power Station.**

The Jatiluhur power station is the first hydroelectric generation of the Citarum basin with a catchment area of 4,607 $km^2$, and its average discharge is 183 $m^3/s$. For exploiting the potential of this cascade, a rockfill dam was constructed with a height of 105 m, a crest length of 1,200 m and a volume of 9.1 $hm^3$. The effective storage capacity is 3,290 $hm^3$ with a water level elevation of 107 m and a low water level operation of 75 m for hydroelectric power, although 47.5 m is for irrigation and water supply to Jakarta city. The hydroelectric power station has been in service since 1965 and its installed capacity is (6x25) MW with a maximal discharge of 324 $m^3/s$.

As follows for simplification, we only deal with the management of Saguling hydroelectric reservoir.

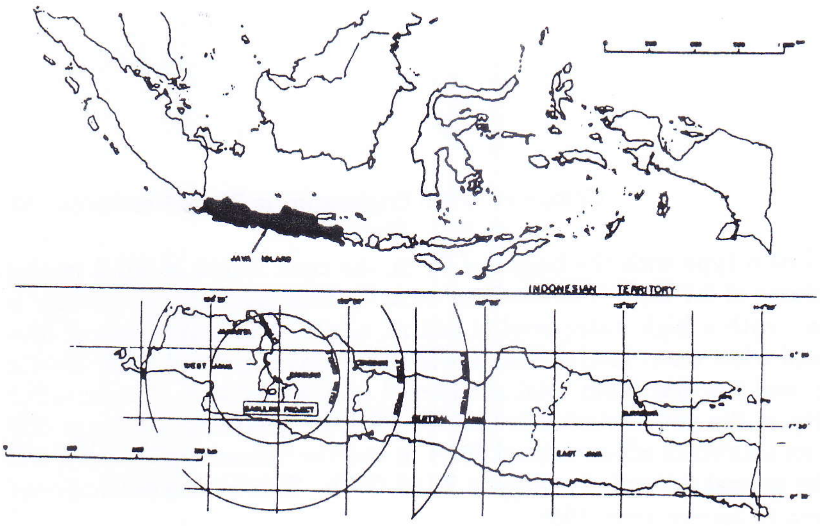


Fig. 1: General geographic situation of the CITARUM BASIN-SAGULING.

## 3. BRIEF REVIEW OF THE OPTIMIZATION OF BELLMAN'S METHOD.

The above method may be thought simple and look naive to the experts, but in order to be fully understood by the readers, we will state precisely our objective. Disposing a constructed data reservoir and a considered information as deterministic upon the discharged inputs a(t) (indeed, a strongly debatable hypothesis). What is managed is to determined the evolution of volume $\Omega$ of the water stock in the reservoir in course of time (in other words the trajectory $\Omega(t)$ )between the volume $\Omega(o)$ on the date of the departure t=0 and the volume $\Omega(T)$ on the date of the arrival t=T, so as to optimize a certain economic function.

This economic function can be the quantity of electric energy production, the amount of sales or that of the benefit on the T period. It can also be the improvement of the agricultural production due to the watering into irrigated perimeters.

For example, if we indicate through $c(t,\Omega,q)$ the economic utility of water in cubic meters with q(t) delivered discharge downstream of the reservoir, the economic optimum can consist in maximizing the sum:

$$S = \int_0^T c(t, \Omega, q)\, q(t)\, dt$$

As long as the reservoir is neither full nor empty, the discharge of exploitation q(t) is given by the relation:

$$q(t) = a(t) - d\Omega/dt$$

If the reservoir is full and if the discharged input has to be very important, the useful discharge can be inferior to the discharged input, and in fact the water is spilled out for nothing; this constitutes a nonlinear constraint, difficult to manage in an approach through analytical calculation but is a good integration in the Bellman's method.

In principle, the optimization has an object for maximizing the functional expression S, dependent on the natural variables, that is time t, and on the state variables (of the control variables), that is volume.

The method of Bellman can be characterized essentially by a double discretization: one of which is time and the other is volume. Thus for passing $\Omega(O)$ to $\Omega(T)$, we will have a definite number of crossing combinations (the number can be very great if the time discretization and volume of the stock are very fine). The period of O to T is divided in n intervals $\Delta t_i$, which a priori do not need to be inevitably matched, but which in practice are often constant and equals for $\Delta t = T/n$. The useful volume of the reservoir $\Omega$max. is also divided in m intervals $\Delta\Omega j$ and they are also inevitably not equal, but for reasons of convenience in particular, more often they are for the calculation of the useful discharge.

To illustrate the purpose, Figure 2 shows a simple example where for passing point A to point B, we will have $5^3$ combinations or possible trajectories. We will sort out in all these trajectories which gives the greater value to the resulting expression from the discretization:

$$\hat{S} = \sum_{i=1}^{n} c(t_i, \Omega_i, q_i)\, q_i \Delta t_i$$

We really account ourselves for the preliminary calculation of the sorting with a fine grid which will discourage the best disposed manual calculator

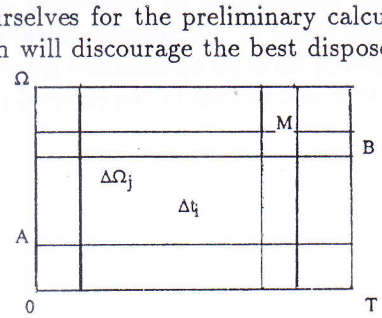


Fig. 2a: Irregular grid.

and which will soon encumber a computer with the same power.

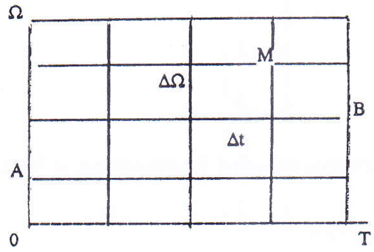


Fig. 2b: Regular grid.

One of the questions that we will consider in a way empirical, but which remains academic, is that of the fineness influence of the cutting out times and volumes.

As it is the matter of a variation problem, the right optimum will only be obtained for a fineness discretization giving the continuous illusion. All the acheived optimums for grid inevitable discretes will be imperfect. It attached a quantity of information in every grid (that can be characterized through an entropy). Intuition tells us that the acheived optimum itself will get better if we refine the grid, that means if we increase that information, but beyond a certain fineness the problems of stability can appear, wherein we will try to detect empirically on the examined practice example.

The use of the sorting or research of the optimal development is very helpful in the principle statement of Bellman, who said that every part of an optimal trajectory is optimal itself. What is done successively in the progress of time between the departure point A and a point course M susceptible to be straight to the optimal trajectory end at B, finally we only take one optimal trajectory. Thus in the progress of time, to every time $t_i$, we only guard m+1 trajectories $\Omega(t)$. The sorting is thus made progressively. An excellently arranged way.

The great advantage of Bellman's method is to make it the best for nonlinear conditions. In our example, we will not misuse this possibility, but the nonlinearities will appear with the sills that constitute the empty reservoir or the spilled out water dam as with the effect of saturation of the utility function.

In connection with the intervention of this in the discretized economic function S, we can refine the presentation in conducting a weighting on the interval $\Delta t_i$ precedent the times $t_i$. As follows, always by simplification, we do not use this weighting through approached integration and we will consider the utility function as constant in the interval $(\Delta t_i, \Delta\Omega_i)$.

Before we start to present the simulation of numeric results according to the Bellman's method, it remains to define the comparison solutions which allow us to appreciate the procured gain through the optimization. The external characterized constraint through the utility function is going to be, unless you hear to the contrary, the same in all treated cases. For lack of precise specific information about the economic situation in Indonesia, it has been constructed from the proposed graph by Mr. BOURGUEIL in a pedagogic demonstration. The utility function that we have chosen is represented in Figure 3. It corresponds to the supposed expenses of the electric energy.

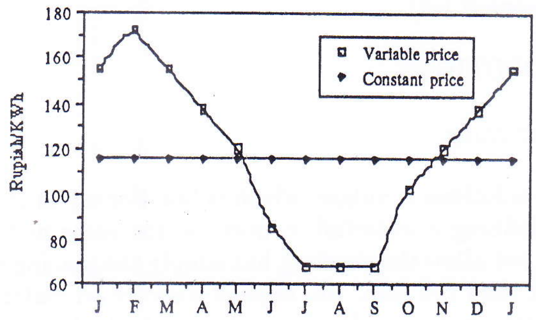


Fig. 3: Price of the electric energy considered as hypothesis.

## 4. THE CHARACTERISTICS OF SAGULING REGULATION.

### 4.1. Hydrology Characteristic.

Table 1 gives the monthly average inflow values to the Saguling reservoir during the period of 1920 to 1980. The discharged inputs represent about two and a half billion cubic meters through the year and can be roughly calculated as four times the effective storage of the Saguling reservoir (600 $hm^3$). The annual average discharged input is 81 $m^3/s$. The highest average monthly discharged input attains 133 $m^3/s$ and the lowest average monthly discharged input in September is only 23 $m^3/s$. We will note that the coefficient of monthly variation is relatively important. It attains 80% in the month of September. This coefficient stays at the height of 20% every year. Meanwhile in all that follows, we are only going to suppose the recurring identical hydrologic years. It goes without saying that we realize the caricature of the reality in that fixed price.

| Months | J | F | M | A | J | J | A | S | O | N | D | J | Mean |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Inflows | 120 | 118 | 133 | 131 | 89 | 52 | 33 | 25 | 23 | 44 | 88 | 123 | 81.4 |
| $\sigma$ | 37 | 40 | 49 | 46 | 43 | 26 | 25 | 19 | 18 | 33 | 42 | 47 | 16.3 |

Table 1: Average monthly inflow of Saguling reservoir ($m^3/s$)

4.2 Economic function of the central hydroelecric power station.

We only consider water used for electricity production. According to the characteristic of the hydroelectric equipment, the approximate expression of the produced power is in megawatts.

$$P = 8.19\ 10^{-3}\ q\ (z-252-1.3\ 10^{-3}\ q^{1.85})$$

where:
q= turbine flow ($m^3/s$)
z= water level in reservoir (m)

## 5. REFERENCE SOLUTIONS.

### 5.1. Streamflow functioning.

One of the reference solutions is vulgar, which is functioning in the absence of a dam. But for defining a waterfall, it must be the same as for a little reservoir which does not allow the stocking but simply the placing in charge. The production, and then the gain, will depend directly on restraining the water level of this little reservoir. We have chosen the high water level of the present constructed reservoir and it is an extremely favourable solution of streamflow functioning. In principle, the existing turbines accecpt a maximal discharge of 228 $m^3/s$.

Two hypotheses of price of the electric energy are going to be envisaged: one of constant price and the other follows Figure 3, inspired by a proposed exercise by Mr. BOURGUEIL from DTG EDF Grenoble (It wasn't possible to obtain exact information on the utility function employed in Indonesia).

The prices concerning the annual gain are given in account units to a certain moment in the Indonesian money, rupiah. The account unit used in this report is approximately equivalent to 17 million rupiahs or 9 thousand dollars (we do not have to dwell with the ephemeral character of this equivalence by holding calculation of the exchange rates changeability).

In the table of results we note with surprise that for streamflow functioning the gain is clearly higher in variable price than in the constant price although this constant price is the annual variable average price. This simply holds a nonlinearity of the average expression. Here the turbine flow q (t) is equal to the inflow average a(t)=81.4 $m^3/s$.

The variable price c(t) can be expressed in function of the constant price equal to the average co and the deviation c(t)-co=$\Delta$c(t).

The gain is then:

$$\int_0^T c(t)\, a(t)\, dt = co\ \bar{a} + \int_0^T \Delta\, c(t)\, .\, (\, a(t) - \bar{a}\, )\, dt$$

In the examined circumstance, price and inflow level are rather in phase, where a complementary term due to deviation product is clearly positive. This is already a sign where the chosen optimization example does not have to bring a sensitive improvement, but as indicated in the introduction, we will take an interest in the effects of discretization which will be perceptible all the same.

5.2. Functioning with dam to constant turbine flow.

This is an interesting solution for the regular supply of electric energy but this is not an advantage from the economic point of view. The engagement in calculation of the variation level in the reservoir tied to the stocking in the moment of strong inflows and to the emptying at low inflows, does not bring a sensitive variation on the gain when a variable price of energy intervenes throughout the year. The loss of the order of 28 account units with the monthly discretization, of the order of 0.2 %, is small and not really significant in comparison with differences due to the base hypothesis (from the consideration of an average hydrologic year).

## 6. APPLICATION OF BELLMAN'S METHOD.

Several critical exercises have been realized:

a) study of the influence of fineness discretization in the times $\Delta$t and the volume $\Delta\Omega$.
b) examination of the effect of a phase displacement more marked between the inflows and the graph of price level.
c) effect of choice of the departure level (equal to the arrival level) and of the tendency towards the optimal stationary regime.

In each critical trial, we took an interest in the difference with a considered case reference.

The underlying idea together with the critical study is to release an expert's report on the opportunity of volume of numerical calculation in the research of optimal solution. In effect we frequently multiply the calculations to end at a result which appears very precise, but where in definitive the "mathematical" precision does not resist before the absences of exactness due to the hypotheses of the simplification (which is here the periodic repetition of an average hydrologic year) or the uncertanties of the base information (definition of the utility function).

## 7. INFLUENCE OF DISCRETIZATION IN TIME AND IN VOLUME.

The steps of $\Delta t$ and $\Delta\ \Omega$ are obviously bound, at least vaguely by the intermediary of the input discharge. On fixed time steps, the possible variations of stocked volume $\Delta\ \Omega$ must be in relation with the size of input volume a $\Delta t$. Of course what arises basically, is (a-q) $\Delta t$; if $\Delta\Omega$ is very small in comparison with a $\Delta t$, it does not create a principle problem but a waste of calculation times. On the other hand great $\Delta\ \Omega$ in comparison with differences (a-q)$\Delta t$ carrying leaps which will destroy the harmonious evolution of Bellman's trajectory, that is the graph $\Omega(t)$.

Departing from an existing work, the question in general is very delimited because the used volume of reservoir is by definition strongly bound to the variations of input discharge.

One of us called to mind in one report (THIRRIOT 1990) that it has not been absolutely indispensable that the input discharge is counted for multiple $\Delta\ \Omega$. Meanwhile for simplifying, we will adopt this convention here. Then it goes without saying that for describing the input flows correctly over a certain duration of times $\Delta t$, it has to use an increment $\Delta\ \Omega$ which is a fraction of these cumulated input flows on $\Delta t$, it means $\Delta\ \Omega = \Delta a\ \Delta t$.

Passing a fine discretization to a rough discretization of time, the number of possible discrete values diminish and it can be difficult to ensure the exact verification of the constant input flow balance in reality and certain instabilities can appear. This precaution explains certain divergences that we can establish later on.

Meanwhile if we stay in sum constant inputs annually, the variational nature of Bellman's method makes us approaching accordingly better to the optimum where the discretization is fine. Departing from this principle, we can consider to proceed with an extrapolation from mean discretization (not too rough) for obtaining an idea of the better value and see if the difference is worth the stake of a more heavy calculation.

Setting to work these directing ideas for the particular case of the Saguling reservoir and for a fixed phase displacement of the price level graph upon the inputs, we obtained the graphs giving the optimal gain in function of

$\Delta t$ (half month, one month, two months, three months) and of $\Delta\ \Omega$ (10, 25, 50, 100 and 200 Million $m^3$).

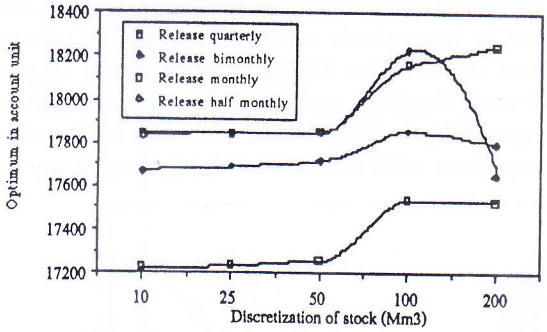


Fig. 4: Influence of time and volume discretization on the instability risks.

We have received some surprises in comparison with the forecasts. In general the maximum gain increases while the time steps decrease, that is not the same with the decreasing of volume steps $\Delta\ \Omega$. In contrary effect to our anticipation, we think that the gain maximum will increase if $\Delta\ \Omega$ increases. This increasing is very weak for $\Delta\ \Omega < 50\ hm^3$ but all the same perceptible. It becomes a slope for the rough values of $\Delta\ \Omega$. We will assign that fact to the effect of monthly inflows discretization. We worked of course through sum constant inputs annually, but the raised inputs conjugation because of the rough discretization and the strong price carrying these gains is apparently too optimistic and must then be distrusted in a summary study.

Except in the case of very rough discretization of maximal volume in three ($\Delta\ \Omega = 200\ hm^3$), the results obtained for monthly discretization and half monthly discretization confounded, this has incited one to limit the volume of numerical calculation (but this saturation effect is maybe a proof that the graph of electricity tarifs did not refine to the half month scale).

We already obtained the results with time steps of two months which make progress regularly and show, for large volume step, the gain over-estimate quite tolerable because of the roughness of the price and hydrology hypotheses.

## 8. EFFECT OF CHOICE OF THE DEPARTURE LEVEL.

The Bellman's method permits us to define a walking optimal $\Omega(t)$ when we settled a point of departure and a point of arrival. In periodic regime the level of stock at the arrival is the same as at the departure. But there was more, in periodic stationary regime, there exists a most optimal trajectory, the best of all, which produces the more possible large gain ; this optimal way can only be found by trial and error and we would appreciate the effect of a deviation of the departure point $\Omega o$ in comparison with the optimal value $\Omega o$ opt.

To avoid the nonlinear effects of pouring sill or of empty reservoir masking evolution through the optimal trajectory, we placed ourselves in a situation where the optimal departure value has been intermediary. Figure 5 gives an example of realization. We see that Bellman's traces (or optimal developments) tend quickly towards the stationary optimal trajectory. We have tried to characterize this tendency by a relaxation of times in examining the deviation (between the traces of Bellman born in a point of any departure and the optimal trajectory) follows a decreased exponential in function of times. The results are quite encouraging as it is seen in figure 6 where the deviations are carried in logarithmic scale. The practice conclusion with two trials will produce traces which surround the optimal trajectory, that one can be approximately localized and especially give a depth idea of the shearing layer corresponding to the perturbation due to the deviation on the departure in comparison with the optimal initial condition.

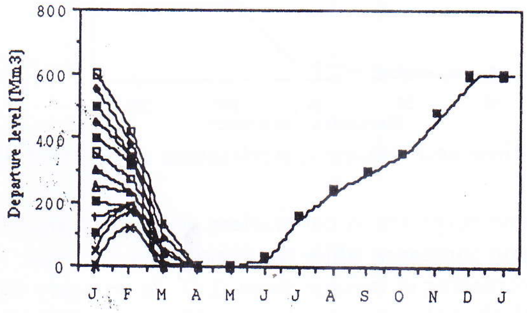


Fig. 5: Towards the stationary optimal trajectory.

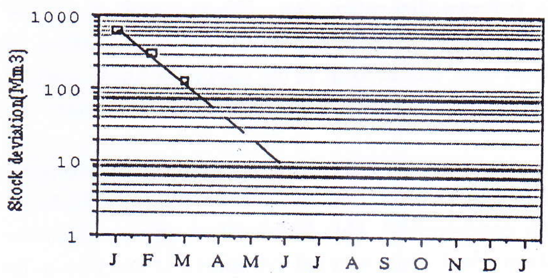


Fig. 6: Times of relaxion towards the stationary optimal trajectory.

## 9. EFFECT OF A PHASE DISPLACEMENT OF THE GRAPH OF PRICE LEVEL.

In the examination of the first results, we noted that the optimization of

Bellman has not brought a sensitive increase of the gain in comparison with functioning streamflow (river flow) to high level.

This holds essentially in reality that the graph of utility function is almost in phase with the graph of input discharge. Meanwhile as our purpose is academic, we would see how Bellman's method has reacted with different phase displacements between hydrologic graph (input flow) and economic graph (price level), Figure 7 shows some of the optimal trajectories so obtained with month by month lag of the price graph. In liason with the previous chapter, it is interesting to note that the majority of optimal departures take place on the sills (empty and full reservoir).

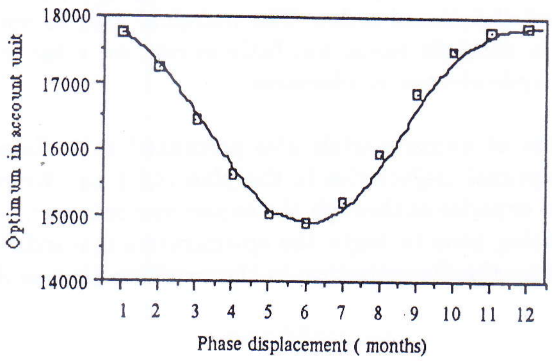


Fig. 7: Effect of a phase displacement of the graph of price level upon the optimum value.

Table 2 shows the comparison of Bellman's solutions with the river flow solution and the turbine flow constant solution (only sensitive to the phase displacement through the intermediary level in the reservoir).

| Displacement (months) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Management | Benefit in account unit | | | | | | | | | | | | |
| Streamflow | 17132 | 16723 | 15861 | 14740 | 14191 | 13001 | 12860 | 13245 | 14074 | 15210 | 16315 | 17003 | |
| Dam | 15574 | 15540 | 15522 | 15525 | 15549 | 15587 | 15629 | 15665 | 15683 | 15680 | 15654 | 15615 | |
| Bellman | 17845 | 17758 | 17238 | 16447 | 15619 | 15027 | 17874 | 15206 | 15943 | 16870 | 17486 | 17790 | |

Table 2: Comparison of gains for the different managements of Saguling reservoir.

## 10. CONCLUSION

The objective which we pursued in this concrete study was to determine better the economic calculation of management optimization of the reservoir in confronting calculation times and safety of the improvement of the management. Of course, from the point of view of particularities of the examined hydroelectric site, there is no question of drawing a universal and definitive conclusion. Meanwhile we have convinced ourselves that beyond a certain discretization fineness for both time and volume, the management has not been sensitive and has become purely a game for universitary workers. In taking the precaution of distributing well the input flow in the given discrete grid by the choice of stock volume of $\Delta\ \Omega$, we can think that the monthly time step is satisfying and that a stock discretization in about ten slices was enough. Time and volume steps two times more rough already give a sufficient indication of the roughness of the basic hypothesis about the postulation of a periodic average hydrology year.

On the other hand, it appears to us essential to extract the proper effect of seasonal variations hydrologic, in which the examined case of the highest basin of Citarum constitutes the preponderant part of the energy gain. In our study, the benefit of the optimization following the Bellman's method only intervenes on the second order. The marginal gain so obtained is indeed important in absolute value, but little in relative value in face of the consequences of hydroelectric randomness.

The multiplication of numeric trials also permitted us to familiarize ourselves with the optimal trajectories in the plan $(t,\Omega)$ and we realized that we obtain helpful experience through the expert systems way, encircling directly the interesting zone to begin the optimization research with a loose grid and then refine the discretization in the sensitive zone so delimited.

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